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MATHEMATICS.—*On Fermat's Last Theorem*, II.[1]

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In a previous communication[2] it has been shown that if x, y, z, be a solution in relatively prime integers, all different from zero, of

$$x^\lambda + y^\lambda = z^\lambda \quad \ldots\ldots (2)$$

$x < y < z$, λ an odd prime > 3, and $xyz \not\equiv 0 \pmod{\lambda}$, then we have

$$x = \alpha\,\xi \quad \alpha^\lambda = z - y \quad \xi^\lambda = \frac{z^\lambda - y^\lambda}{z - y} \quad \ldots\ldots (3)$$

$$y = \beta\,\eta \quad \beta^\lambda = z - x \quad \eta^\lambda = \frac{z^\lambda - x^\lambda}{z - x} \quad \ldots\ldots (4)$$

$$z = \gamma\,\zeta \quad \gamma^\lambda = x + y \quad \zeta^\lambda = \frac{x^\lambda + y^\lambda}{x + y} \quad \ldots\ldots (5)$$

in which α, β, γ, ξ, η, ζ, λ, are all relatively prime integers; and

$$\alpha + \beta = \gamma \pm \delta \quad \ldots\ldots (19)$$

[1] Received February 19, 1929.

[2] THIS JOURNAL 18: 389. 1928. The following typographical errors should be noted: Page 390, equation (4_3), for y read x; last line, for "relative primes," read "relatively prime." Page 391, last term of (9 *bis*), for z read ζ. Page 392, eq. (19), for Δ read M. Page 394, for $\xi < \eta < \zeta$, read $\xi > \eta > \zeta$; for $H = \eta/\xi$, read $H = \eta/\xi$. On page 395, eq. (25) should be numbered (35), and (26) should be (36); the first equation on page 395 should be (34), and the last (37); the next to the last equation on page 394 should be numbered (33); and on page 395, line seven, for "by (24) and (25)" read "by (24) and (35)."

i.e., $\gamma \neq \alpha + \beta$, where δ is some positive integer > 0. The following inequalities were also proved:

$$2\lambda^2 < \delta < \alpha < \beta < \gamma \quad \ldots\ldots (39)^3$$

$$\xi > \eta > \zeta \quad \ldots\ldots (40)$$

$$\lambda \alpha^{\lambda} < x \quad \ldots\ldots (35)$$

$$\alpha + \delta < 2\alpha \left(\frac{\eta}{\xi}\right)^{\lambda - 1} \quad \text{if} \quad \alpha + \beta + \delta = \gamma \quad \ldots\ldots (33)$$

$$\xi^{\lambda} < \lambda z^{\lambda - 1} \quad \ldots\ldots (30)$$

$$\lambda x^{\lambda - 1} < \lambda y^{\lambda - 1} < \xi^{\lambda} \quad \ldots\ldots (34)$$

From (30) and (34)

$$\lambda y^{\lambda - 1} < \xi^{\lambda} < z^{\lambda - 1} \lambda \quad \ldots\ldots (41)$$

Also

$$\lambda x^{\lambda - 1} < \eta^{\lambda} < z^{\lambda - 1} \lambda \quad \ldots\ldots (42)$$

the proof that $\eta^{\lambda} > \lambda x^{\lambda - 1}$ being similar to that of (30).
Furthermore, putting

$$d = x + y - z \quad \ldots\ldots (9\ bis)$$

we have $d < x$, since $y - z$ is negative, and hence

$$d < x < y < z \quad \ldots\ldots (38)$$

Also, by pairing successive terms in the expansion of (5_3), $x < y$, λ odd, we see that

$$x^{\lambda - 1} < \zeta^{\lambda} < y^{\lambda - 1} \quad \ldots\ldots (43)$$

Now, writing (19) in the form

$$\gamma = \frac{\gamma^{\lambda}}{\gamma^{\lambda - 1}} = \alpha + \beta \pm \delta$$

[3] Abel's Theorem that $x^{\lambda} + y^{\lambda} = z^{\lambda}$ has no solution in integers if x, y, or z be prime was proved by E. Lucas; but Markoff, L'interméd. de math. **2**: 23. 1895, and **8**: 305. 1901, pointed out that Lucas' proof is incomplete, as the case $z = y + 1$, i.e., $\alpha = 1$, $x = \xi$, is not included. Markoff asked if it is possible for $(y + 1)^{\lambda} = x^{\lambda} + y^{\lambda}$; eq. (39) shows that this is impossible, for $\alpha \neq 1$ if $2\lambda^2 < \alpha$.

and substituting for γ from (5_2), we have by (3_1) and (4_1)

$$\alpha\,\xi + \beta\,\eta = \gamma^{\lambda-1}\,(\alpha + \beta \pm \delta)$$

whence

$$\alpha\,(\gamma^{\lambda-1} - \xi) + \beta\,(\gamma^{\lambda-1} - \eta) \pm \delta\,\gamma^{\lambda-1} = 0 \ldots\ldots (44)$$

If

$$\xi < \gamma^{\lambda-1} \ldots\ldots (45)$$

then, by (40), $\eta < \gamma^{\lambda-1}$, and the ambiguous sign in (44) is negative; but if this sign be negative, then at least one of the other terms must be positive and therefore either $\eta < \xi < \gamma^{\lambda-1}$ as before, or else $\eta < \gamma^{\lambda-1} < \xi$. Similarly, if

$$\gamma^{\lambda-1} < \xi \ldots\ldots (46)$$

then either $\eta < \gamma^{\lambda-1} < \xi$, in which case the sign remains ambiguous, or else $\gamma^{\lambda-1} < \eta < \xi$ when the sign must be positive; and if the sign be positive, we may have either $\gamma^{\lambda-1} < \eta < \xi$ or $\eta < \gamma^{\lambda-1} < \xi$. Altogether, there exist the following possibilities:

$$\eta < \xi < \gamma^{\lambda-1} \quad \gamma = \alpha + \beta - \delta \ldots\ldots (A)$$

$$\eta < \gamma^{\lambda-1} < \xi \quad \gamma = \alpha + \beta - \delta \ldots\ldots (B)$$

$$\eta < \gamma^{\lambda-1} < \xi \quad \gamma = \alpha + \beta + \delta \ldots\ldots (C)$$

$$\gamma^{\lambda-1} < \eta < \xi \quad \gamma = \alpha + \beta + \delta \ldots\ldots (D)$$

The inequality (45) is a *sufficient* condition that the ambiguous sign in (44) be negative, and hence that $\gamma = \alpha + \beta - \delta$, while (46) is a *necessary* condition that it be positive and $\gamma = \alpha + \beta + \delta$.

From (41) and (46), $\gamma^{\lambda(\lambda-1)} < z^{\lambda-1}\,\lambda$, whence $\gamma^{\lambda} < z\,\sqrt[\lambda-1]{\lambda}$ or, since $\gamma^{\lambda} = x + y = z + d$, $z + d < z\,\sqrt[\lambda-1]{\lambda}$, and therefore if $\gamma^{\lambda-1} < \xi$,

$$d < z\left(\sqrt[\lambda-1]{\lambda} - 1\right) \ldots\ldots (47)$$

Subtracting α^{λ} from both members of (35), we get by (3_2) and (9 *bis*)

$$(\lambda - 1)\,\alpha^{\lambda} < d \ldots\ldots (48)$$

Adding $(\lambda - 1)d$ to both members of (48) gives

$$\frac{\lambda - 1}{\lambda}\,x < d \ldots\ldots (49)$$

Hence, by (47) and (49),

$$qx < z \qquad (50)$$

in which

$$q \equiv \frac{\lambda - 1}{\lambda \left(\sqrt[\lambda - 1]{\lambda} - 1 \right)} \qquad (51)$$

Calculation shows that for every $\lambda > 5$, $q > 2$ (λ odd). Obviously, therefore, if $\gamma^{\lambda - 1} < \xi$,

$$2x < z \qquad (52)$$

whence by (4_2)

$$x < \beta^{\lambda} \qquad (53)$$

from which by (38)

$$d < \beta^{\lambda} \qquad (54)$$

or by (9 *bis*)

$$y < 2\beta^{\lambda} \qquad (55)$$

Now, if $\alpha + \beta + \delta = \gamma$, we have by (33)

$$\xi < \eta \left(\frac{2\alpha}{\alpha + \delta} \right)^{\frac{1}{\lambda - 1}} \qquad (56)$$

whence by (41)

$$\lambda y^{\lambda - 1} < \xi^{\lambda} < \eta^{\lambda} \left(\frac{2\alpha}{\alpha + \delta} \right)^{\frac{\lambda}{\lambda - 1}} \qquad (57)$$

Multiplying by y ($= \beta\eta$), we get from (55)

$$\lambda \beta^{\lambda} < y \left(\frac{2\alpha}{\alpha + \delta} \right)^{\frac{\lambda}{\lambda - 1}} < 2\beta^{\lambda} \left(\frac{2\alpha}{\alpha + \delta} \right)^{\frac{\lambda}{\lambda - 1}},$$

$$\lambda < 4 \left(2^{\frac{1}{\lambda - 1}} \right) \left(\frac{\alpha}{\alpha + \delta} \right)^{\frac{\lambda}{\lambda - 1}}$$

which shows that if $\gamma^{\lambda - 1} < \xi$, and $\gamma = \alpha + \beta + \delta$, then

$$\lambda < 4 \sqrt[\lambda - 1]{2} \qquad (58)$$

and it would then be impossible to have $\lambda > 3$. The relations (C) and (D) are therefore ruled out; and

$$\gamma \neq \alpha + \beta + \delta \quad \text{.....(25 bis)}$$

By raising both members of (50) to the λth power and subtracting x^λ, we see that if $\gamma^{\lambda-1} < \xi$,

$$(q^\lambda - 1)\, x^\lambda < y^\lambda \quad \text{.....(59)}$$

Adding the positive quantity $(\gamma^\lambda - 1)y^\lambda$, we get

$$(q^\lambda - 1)\, z^\lambda < q^\lambda\, y^\lambda$$

or

$$z < y \sqrt[\lambda]{1 + \frac{1}{q^\lambda - 1}}$$

and writing the absolutely convergent binomial series for the root in the form

$$\left(1 + \frac{1}{\lambda\,(q^\lambda - 1)}\right) + \frac{1}{\lambda}\left(\frac{1-\lambda}{2\lambda}\right)\left(\frac{1}{q^\lambda - 1}\right)^2\left[1 + \frac{1-2\lambda}{3\lambda}\left(\frac{1}{q^\lambda - 1}\right)\right] +$$

$$\frac{1}{\lambda}\left(\frac{1-\lambda}{2\lambda}\right)\left(\frac{1-2\lambda}{3\lambda}\right)\left(\frac{1-3\lambda}{4\lambda}\right)\left(\frac{1}{q^\lambda - 1}\right)^4\left[1 + \frac{1-4\lambda}{5\lambda}\left(\frac{1}{q^\lambda - 1}\right)\right] + \ldots$$

we see, $y < z$, $\lambda > 3$, that when $\gamma^{\lambda-1} < \xi$,

$$1 < \frac{z}{y} < \left[1 + \frac{1}{\lambda\,(q^\lambda - 1)}\right] \quad \text{.....(60)}$$

The extreme right member approaches the value unity so rapidly with increasing λ, however,[4] as to make the supposition $\gamma^{\lambda-1} < \xi$ apparently untenable; this rules out relation (B).

[4] Dickson, Quar. Jour. Math. **40**: 27–45, 1908, proved Fermat's Theorem for $n < 7000$; and Beeger, Mess. Math. (2), **55**: 17–26, 1925, showed that $x^\lambda + y^\lambda + z^\lambda = 0$ has no solution in integers prime to λ for $\lambda < 14000$. For $\lambda = 7001$ (the next prime above 7000), $q > 769$, and $1 < z/y < 1 + 10^{-20209}$; for $\lambda = 14009$, $q > 1428$, and $1 < z/y < 1 + 10^{-40131}$. Again, from (30), (46), and (5_1), we have $\xi^\lambda < \lambda\, z^{\lambda-1} < \lambda\, \xi\, \zeta^{\lambda-1}$, or $\xi < \zeta \sqrt[\lambda-1]{\lambda}$; while by ($43_3$) and ($34_2$), $\lambda\, \zeta^\lambda < \lambda y^{\lambda-1} < \xi^\lambda$ or $\zeta \sqrt[\lambda]{\lambda} < \xi$; combining these two results, $\sqrt[\lambda]{\lambda} < \frac{\xi}{\zeta} < \sqrt[\lambda-1]{\lambda}$, or $1 < \frac{\xi}{\zeta \sqrt[\lambda]{\lambda}} < \sqrt[\lambda(\lambda-1)]{\lambda}$, where for $\lambda = 14009$, $\lambda\,(\lambda-1) = 196,238,072$.

It may be noted that, by (60),

$$\alpha^\lambda \lambda (q^\lambda - 1) < y < z \quad \ldots\ldots\ldots\ldots\ldots\ldots (61)$$

If $2x < z$, then by (40), $2\alpha < \gamma\left(\frac{\zeta}{\xi}\right) < \gamma$, i.e., $2\alpha < \gamma$; it also follows from (46) that $\gamma^\lambda < \gamma\xi$, whence $x + y < x + (\gamma - \alpha)\ \xi$ or $(\alpha\ \xi =)\ x < y < \xi\ (\gamma - \alpha)$ or $2\alpha < \gamma$. If $2\alpha > \gamma$, then, multiplying by $\sqrt[\lambda]{\lambda}$, we get from $(3_1), (5_1)$, and the result $\zeta\sqrt[\lambda]{\lambda} < \xi$, that $z\sqrt[\lambda]{\lambda} < 2x$ or $\frac{2x}{z} > \sqrt[\lambda]{\lambda} > 1$, whence $\gamma < 2\alpha$ only if $2x > z$, which in turn is possible, by (52), only if (45) is satisfied.

From (9 *bis*),

$$\left.\begin{aligned} \xi &= \alpha^{\lambda-1} + \frac{d}{\alpha} \\ \eta &= \beta^{\lambda-1} + \frac{d}{\beta} \\ \zeta &= \gamma^{\lambda-1} - \frac{d}{\gamma} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (62)$$

Substituting (9 bis_2) for $\beta^{\lambda-1}$ in (41_1), we get $\lambda\eta^{\lambda-1}\left(\eta - \frac{d}{\beta}\right) < \xi^\lambda$ or $\lambda\eta^\lambda < \xi^\lambda + \frac{\lambda d}{\beta}\eta^{\lambda-1}$; whence, if $\xi^\lambda < \frac{\lambda d}{\beta}\eta^{\lambda-1}$, then $y < 2d$, while if $\xi^\lambda > \frac{\lambda d}{\beta}\eta^{\lambda-1}$ then $\lambda\eta^\lambda < 2\xi^\lambda$. In the same way, we find from (43_2) that either $\zeta^\lambda < \frac{d}{\beta}\eta^{\lambda-1}$ and $2\zeta^\lambda < \eta^\lambda$, or else $\zeta^\lambda > \frac{d}{\beta}\eta^{\lambda-1}$ and $2d < y$. From (30), $\xi^\lambda < \lambda z^{\lambda-1} = \lambda\zeta^{\lambda-1}\left(\zeta + \frac{d}{\gamma}\right)$ or, by (38), $\xi < \zeta\sqrt[\lambda]{2\lambda}$. If we suppose

$$\left.\begin{aligned} &\xi^\lambda > \frac{\lambda\, d}{\beta}\eta^{\lambda-1},\ \lambda\,\eta^\lambda < 2\,\xi^{\lambda-1} \\ &\zeta^\lambda > \frac{d}{\beta}\eta^{\lambda-1},\ 2\,d < y \\ &\xi^\lambda < 2\,\lambda\,\zeta^\lambda \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (63)$$

then from the last, and the second part of the first, of these equations, we get $\eta^\lambda < 4\zeta$; but from (2) and (38), $z^\lambda < 2y^\lambda$ or $\zeta^\lambda < \frac{2y^\lambda}{\gamma^\lambda}$; consequently, $\gamma^\lambda < 8\beta^\lambda$, and thus by (3_2) and (4_2), $7x < y < z$, or $2x < z$, and our supposition is not tenable if $z < 2x$. If, however, we suppose

$$\left.\begin{aligned} \xi^\lambda &< \frac{\lambda\, d}{\beta}\,\eta^{\lambda-1},\ y < 2d \\ \zeta^\lambda &< \frac{d}{\beta}\,\eta^{\lambda-1},\ 2\,\zeta^\lambda < \eta^\lambda \\ \xi^\lambda &< 2\,\lambda\,\zeta^\lambda \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (64)$$

then, if $\xi < \gamma^{\lambda-1}$ and $z < 2x$, we see that if $y < 2d = \gamma^\lambda - \beta^\lambda - \alpha^\lambda$ then $\beta^\lambda < d$ and $\alpha^\lambda + \beta^\lambda < x < y < z$. By (39), $\delta < \alpha$; and in the first paper it has been shown that δ contains the factors 6 and λ^2; whence $6\lambda^2 < \alpha$, and

$$[2\,(6\,\lambda^2)^\lambda]^\lambda < [\alpha^\lambda + \beta^\lambda]^\lambda < z^\lambda = x^\lambda + y^\lambda \ldots\ldots\ldots (65)$$

From the last, and the second part of the second, of these three equations, we get $\xi < \eta\,\sqrt[\lambda]{\lambda}$. By (4_2), (5_1), and (23), if $\gamma = \alpha + \beta - \delta$,

$$x + \beta^\lambda = z = (\alpha + \beta - \delta)\,\zeta \ldots\ldots\ldots\ldots (66)$$

$$\alpha\,(\xi - \zeta) = \beta\,(\zeta - \beta^{\lambda-1}) - \delta\,\zeta \ldots\ldots\ldots\ldots (67)$$

where, since $\zeta < \xi$, $\delta\zeta < \beta\,(\zeta - \beta^{\lambda-1})$,

$$\beta^{\lambda-1} < \zeta \ldots\ldots\ldots\ldots\ldots\ldots\ldots (68)$$

Collecting our results,

$$\alpha^{\lambda-1} < \beta^{\lambda-1} < \zeta < \eta < \xi < \gamma^{\lambda-1} \ldots\ldots\ldots\ldots (69)$$

Hence we have in general

$$\xi < \gamma^{\lambda-1},\ \gamma = \alpha + \beta - \delta \ldots\ldots\ldots\ldots (70)$$

δ a positive integer.

In a later paper, the decomposition of α, β, γ, δ, into factors will be considered.

PHYSICS.—*The stratified settling of fine sediments.*[1] P. G. NUTTING, U. S. Geological Survey.

Muddy water settles either clear to a false bottom or cloudy to an indefinite bottom. Clear settling indicates impure water, either salts in solution or a second colloid in proper proportion, while cloudy settling is generally proof of very pure water. The water of a lake having a false bottom is certainly contaminated to some extent, however clear, but the water of a lake or river that remains turbid after days of settling is free from salts in solution, bacteria and organic colloids since these, if ever present, have all been fixed by excess clay colloids. Extremes of concentration, valency, ionization, adsorption, particle fineness, dissolved air and many other factors may vary the picture somewhat but the rough sketch is sufficient introduction to the subject of this paper.

The chemical literature bearing on the precipitation of colloids is very extensive and has been ably summarized in the well known works of Bancroft, Alexander, Svedberg, Freundlich, Weiser and others. Physicists however have done very little with their side of the problems involved in the precipitation of fine particles though few phenomena in physics are more fascinating or instructive than the play of these extremely gentle forces, very delicately balanced, with the minute but violent Brownian movement ever present.

Carl Barus, of this laboratory,[2] made many observations on stratified settling but considered only gravity, particle size and viscosity of fluid among the physical factors involved, ignoring upward diffusion under the Brownian movement. Einstein[3] developed a statistical theory of the Brownian movement which gave the equilibrium distribution of one size of particles suspended in a liquid but his equations are too cumbersome to deal with a wide range of sizes. Perrin[4] verified Einstein's theory of stable distribution by actual counting and measurement. Mendenhall and Mason[5] reached the conclusion that stratified settling is due to minute temperature differences causing weak convection currents—a conclusion not acceptable to many thoroughly familiar with the behavior of fine suspensions. This paper is to present a theory much simpler and more general than Einstein's,

[1] Published by permission of the Director, U. S. Geological Survey. Received Sept. 28, 1929.

[2] BARUS. U.S.G.S. Bull. 36. 1886.

[3] EINSTEIN. Ann. Physik 17: 549. 1905.

[4] PERRIN. Comptes rend. 146: 968. 1908.

[5] MENDENHALL and MASON. Proc. Nat. Acad. Sci. 9: 199, 202. 1923.

which gives Perrin's law of distribution as a special case and even covers stratified settling quite adequately. It is incidental to a study of the physico-chemical relations between the three hydrous oxides, silica (acid), alumina (amphoteric), and ferric hydroxide (basic) and the bentonites and other clays.

Stratified suspensions are readily prepared with almost any kind of fine particles shaken up with distilled water. Should they settle clear at first, repeated washing and settling will eventually bring about a condition where finer material will be left behind in suspension. After an hour or two this begins to settle away from the surface leaving a still finer suspension above it. Eventually five or more sharply distinct layers may be in the process of settling at the same time. Barus (*l.c.* 1886) made precise measurements of the rates of fall of many of these layers. Hydrated alumina and ferric oxide, chemically prepared, both flocculate and settle clear after repeated washings but eventually yield and come down in cloudy layers. Sodium silicate, if very highly diluted, appears to hydrolyze and precipitate white particles of silica. Dialyzed iron, diluted with pure water, shows no tendency to settle even after years of standing, yet a very small amount of purified bentonite will bring it down in a thick floc.

In a mass of particles at the absolute temperature T, the thermal or kinetic (distending) pressure p outward due to molecular or Brownian movement is[6]

$$p = CJRT/M \text{ dynes/cm}^2 \tag{1}$$

when C is the concentration in grams/cc, $JR = 8.30 \times 10^7$ and M is molecular weight. For example, in a suspension containing 1 gram per liter of particles each consisting of 10,000 molecules each of molecular weight 100, the pressure at 300° absolute is 24.9 dynes/cm²—a very gentle pressure in comparison with the kinetic pressure in water, 1380×10^6 dynes/cm², on account of the size of the particles and the sluggishness of their movements.

The pull downward by gravity on such an aggregate of particles is

$$w = Ckg \text{ dynes/cc} \tag{2}$$

where $k = 1 - \rho_l/\rho_g\, g = 980$ dynes/gram and ρ_l and ρ_g are the densities of liquid and of suspended grains.

The pressure gradient, dp/dz, depends on the ratio CT/M. If this is constant, the kinetic pressure is uniform and the gradient zero.

[6] NUTTING. This JOURNAL 19: 298. 1929.

In a true solution C and M are both constant. Even in a suspension of fine particles, CT/M may be constant if the concentration be proportional to size of particle, i.e., if the number of particles per cc is constant from top to bottom or throughout any given layer. In general an aggregate of particles will be in equilibrium distribution if the pressure gradient upward just equals the weight of suspended material in unit volume,

$$\mathrm{d}p/\mathrm{d}z = \mathrm{d}w \tag{3}$$

or, by substitution from (1) and (2)

$$\frac{\mathrm{d}\log(CT/M)}{\mathrm{d}z} = \frac{Mkg}{JRT} \tag{4}$$

which is the most general equilibrium condition. In the special case of uniform mass of particle M and uniform temperature T, (4) r-e duces to d log $C/dz = Mkg/JRT$ which may be integrated giving Perrin's equation which may be written

$$\log(C/C_0) = Mkgz/JRT \tag{5}$$

where z is the depth measured downward from the surface and C_0 is the concentration at the surface.

In the numerical example cited above, 1 mg of suspended material per cc accounted for a kinetic pressure of 24.9 dynes/cm², the weight of that material, $Ckg = 0.56$ dynes/cm²/cm (taking $k = 0.4$) hence for a steady state, the kinetic pressure must increase downward about 2 per cent per cm of depth. Hence CT/M must so increase and since a 2 per cent increase in T or decrease in M is very unlikely, the burden lies chiefly on the concentration C.

If the pressure gradient in a suspension is greater than its immersed weight then it will diffuse upward to the surface, if less than that weight it will settle according to the generalized Stokes' Law with the velocity

$$v = \frac{2\,r^2\,\rho_g}{9\,\eta\,C}\left(Ckg - \frac{JRT}{M}\frac{\mathrm{d}c}{\mathrm{d}z}\right) \tag{6}$$

The expression bracketed is the resultant force acting on all the particles in unit volume. When the concentration gradient $dc/dz = 0$, (6) reduces to the ordinary form of Stokes' Law. It may be put in different forms by using the substitutions $C = mN$, $m = V\rho = M \times$ mass of hydrogen atom 1.66×10^{-24} gram.

From the form of (6) it is easily seen that the velocity of fall is very sensitive to size of particle. It may readily be zero or even negative (upward) for the smaller particles. As each size of particles falls, smaller particles are not only left behind but tend to diffuse upward. But there is a limit to this tendency to separate because it must cease when the concentration gradient dc/dz reaches a certain minimum or the particles fail to make contact. This appears to be the explanation of the sharp upper boundary of particles exceeding a certain size.

In pure water the ionization is too slight and the adsorbed ions too weak to interfere greatly with the diffusion and settling. In salt solutions the reverse is the case, adhesion between particles is promoted by strong adsorbed ions, and coagulation and settling are rapid. However such phenomena have been much discussed by chemists.

Dialyzed iron, diluted with pure water in a clean glass vessel, stays up indefinitely. At the top it can not escape while on the walls and bottom the kinetic pressure is reflected in full force by the clean glass. But if the bottom say is made of clean jelly (such as prepared bentonite), particles are adsorbed instead of reflected when they hit and the suspended iron is gradually taken up.

The effect of what appears to be direct light pressure I have frequently observed when a suspension of extremely fine particles is left for several days in a tube exposed to a strong horizontal light. A large proportion of the finer particles are driven along the light path and are found adhering to the wall where it intersects the light beam and not elsewhere.

The thermal effect to which Mendenhall and Mason (*l.c. ante*) attribute stratified settling appears to be but one of its minor causes. They found that stratification did not occur in a constant temperature room and that strata already formed in a lighted room disappeared when removed to a constant temperature room. I have been unable to confirm their conclusions in a single instance. I find that a properly prepared suspension stratifies as readily in a dark, corstant temperature room as in the open. In Barus' original experiments, described in 1886 (*l.c. ante*) he was particularly careful to guard against convection currents. From the theoretical equation (4) it is seen that the same percentage change in C, T, or M is equally effective against gravity. But a variation (with depth) of say 1 per cent in either C or M is far more likely than in T, where it would amount to 3°C.

SUMMARY

A gradient in kinetic pressure opposes gravity in the settling of fine sediments.

A simple general law is derived for the equilibrium distribution of fine particles in suspension of which Perrin's equation is a special case.

Stokes' Law is given a more general form to include the upward gradient of kinetic pressure as well as gravity as affecting the fall of particles.

The upward diffusion of a given size of particles is limited by the supply, hence the various sizes show a sharp upper limit.

A non-reflecting wall tends to take up a suspension.

Mendenhall and Mason's explanation of stratified settling could not be confirmed experimentally and is shown to be a minor cause in theory.

PALEONTOLOGY.—*New Carboniferous invertebrates*—II.[1] GEORGE H. GIRTY, U. S. Geological Survey.

This paper contains descriptions of seven brachiopods from western Texas.

Schizophoria hueconiana, n. sp.

Figures 29–36

Shell small, transverse, subelliptical in outline.

Pedicle valve strongly transverse, broadly and almost regularly rounded at the sides and more or less straight across the cardinal and anterior margins. The anterior outline, however, is slightly emarginate and the cardinal outline, though essentially straight, is interrupted by the umbonal parts of both valves which project beyond it though not very far. The convexity is rather low and chiefly developed in the posterior half, the anterior half over much of its width being depressed into a broad, fairly deep sinus. Umbonal region rather gibbous. Beak small, pointed, neither strongly incurved nor produced much beyond the hinge. The cardinal line is equal to or nearly equal to one-half the greatest width; scarcely distinguishable in the generally rounded outline. Cardinal area rather low, much wider than high, strongly arched and having a general backward direction of about 145° to the plane of the margin though, because of its curvature, more nearly perpendicular to it in the lower part.

Brachial valve strongly convex, more or less inflated in the umbonal region, and more or less compressed at the sides. Beak small and strongly incurved. The tip of the beak does not project so far beyond the hinge as the parts just in front which are about on a level with the beak of the pedicle valve.

The shell arches strongly from side to side, flaring somewhat near the lateral margins but the curvature is not differentiated into an appreciable

[1] For the previous paper in this series see this JOURNAL 19: 135–142. Published by permission of the Director of the U. S. Geological Survey. Received September 30, 1929.

fold to match the sinus of the other valve. This tends to produce a broad, gentle emargination across the front.

The surface is marked by very fine, sharp, radial lirae, some of which (in the usual manner) gradually rise to a greater prominence than the rest and terminate abruptly in a hollow "spine," represented by a small opening in the shell. These tubular lirae are very small and so little differentiated from the rest that they are apt rather to escape attention than to attract it. Three or four lirae occur in 1 mm. toward the front.

This species might in a sense be regarded as *S. resupinoides* in miniature. It is obviously distinct from that well known Pennsylvanian shell, which nevertheless is the one most comparable to it in our American faunas. It is of course much smaller than *S. resupinoides* and the median sinus in the pedicle valve is much broader. *S. resupinoides*, when of the same size as *S. hueconiana*, has no sinus at all.

The foregoing description is drawn up from the type specimen. My collections contain a number of other specimens but they show little that is additional. Most of them are small and apparently immature and most of them, too, are compressed out of their natural shape. One is as large as 27 mm. in width which is about one-fourth larger than the type. Another is of about the same size as the type and the rest are smaller in varying degrees. About the only part of the foregoing description that needs to be qualified in the light of this other material concerns the sculpture which is better preserved on some of those specimens than on the typical one. The specialized elevated tubular lirae are more conspicuous than they are on the type, but even so the sculpture is developed on so small a scale that the differentiation is only conspicuous under a good lens. These lirae, enlarged and elevated beyond the rest, are in the pedicle valve especially noticeable over the median part, and in the brachial value over the lateral parts, and under a lens they are indeed rather conspicuous. In the immature specimen figured to show this feature, it will be noted that in neither valve is the beak as much incurved as are the beaks of the type specimen. This difference is probably to be attributed to its immature condition as the beaks are commonly more erect in young brachiopods than in old ones, although it may in some degree be an original but individual character and in some degree accidental, due to the compression which the specimen has manifestly suffered.

Horizon and locality: Upper part of the Hueco limestone; Marble canyon, Diablo plateau, Van Horn quadrangle, Texas.

Chonetes quadratus, n. sp.

Figures 4–7

Shell small, highly arched, strongly flexed into median fold and sinus, quadrate in outline, smooth as to surface.

The pedicle valve is transversely subquadrate, scarcely wider at the hinge than in front of it. The sides which are nearly straight and nearly parallel, converge very slightly forward and may be slightly emarginate below the cardinal angles which otherwise vary but little from 90°. The anterior out-

line is broadly and gently concave with abrupt turns where it joins the lateral outlines. The convexity is usually high but it is offset by the unusually broad, deep sinus which is sub-angular at the bottom and, beginning at the beak, deepens and widens rapidly so as to occupy practically the entire anterior margin. The auricles are much depressed and explanate. This combination of characters causes the valve, in other terms, to consist of two high and abruptly rounded ridges that diverge to the two anterio-lateral angles, are separated by the deep broad sinus and descend steeply at the sides to the small explanate auricles. The bases of two small spines are shown on the outer parts of the cardinal area and doubtless several other spines of which there is now no evidence were developed nearer the beak. The surface appears to be quite smooth; it does not even show the small pores or hollow spines which are commonly found in these shells and which must be believed to have been originally present.

Brachial valve unknown.

C. quadratus is more nearly comparable to *C. subliratus* than to any other American *Chonetes* at present known. It is, however, a decidedly smaller shell, is much less extended at the hinge, has a deeper, broader and more angular sinus and is smooth instead of obscurely striated. I have in all five specimens of this species. They vary but little in size and this fact together with their remarkably strong convexity indicates that we are dealing with a species in its mature stage, although the largest is only 11 mm. wide at the hinge. All the specimens are in essential agreement with the decription above formulated. The sides diverge above more in some than in others but *C. quadratus* is not to be compared with *C. subliratus* in this respect any more than it is in size. In the depth and angular shape of the sinus and in the prominent, abruptly rounded ridges between which it lies, *S. quadratus* again is scarcely to be compared with the typical specimen of *C. subliratus*, for in these characters it is as much more, as in the others it is less, pronounced. One of the other figured specimens of *C. subliratus* leans rather more towards *C. quadratus* than the typical one in these last characters, but there is nevertheless a wide difference. If compared with young specimens of *C. subliratus* of equal size, these differences are very marked inasmuch as the convexity is lower and the sinus shallower in young specimens than in old ones. I am not, in this instance, stressing the difference in the surface markings because to some extent the surface markings are the sport of preservation; nevertheless this shell is apparently quite devoid of radial striae whereas *C. subliratus* has fine though somewhat obscure radii. The radial markings may be the product of slight exfoliation but this explanation at present seems scarcely probable.

C. quadratus also recalls the form that Meek described as *C. verneuilianus* var. *utahensis* which seems, however, to be less nearly allied to the present species than to *C. subliratus*. *C. quadratus* consequently differs from it in nearly the same way that it differs from *C. subliratus*—in the less extended hinge, in the deeper, broader, more angular sinus and in the smooth instead of

striated surface. Some doubt may be admitted as to the original markings of *C. subliratus* but v. *utahensis* apparently has the sharply marked lirae of *C. verneuilianus*.

Horizon and locality: Delaware Mountain formation; Victoria peak, Diablo plateau, Van Horn quadrangle, Texas.

Chonetes consanguineus n. sp.

Figures 8–9

Shell of medium size, strongly transverse, semicircular in outline, moderately arched but deflected medially into a fairly strong, narrow fold and sinus, and as to surface essentially smooth or without trace of radial sculpture.

The pedicle valve is semi-circular in outline, much wider than long and widest at the hinge. The outline around the sides and front is an irregular curve, almost straight at the sides above and only gently arched across the front. The sides diverge strongly toward the hinge with which they form acute angles; the cardinal angles may even be slightly extended with the outline below slightly emarginate. The convexity is moderate and is somewhat greater at or in front of the middle than behind it. The umbonal swelling is broad and low passing by degrees into the large, undefined auricles. The sinus is a conspicuous feature being rather deep and rather narrow, though rounded. The cardinal spines are small and numerous; 5 or 6 are indicated on each side of the beak near which there were undoubtedly others that have been destroyed without leaving a trace.

The surface is entirely without radial sculpture and is smooth save for two minor features. Lamellose striae marking stages of growth are fairly numerous but of unequal strength and irregular distribution, while some specimens show the openings left by minute hollow spines or arched scales. These are very numerous and very small; they do not ordinarily interrupt the smooth appearance of the surface but in certain conditions of preservation they give rise to excavations somewhat larger than the pores themselves as if microscopic pieces had been gouged out of the shell.

The brachial valve agrees with the pedicle valve in the usual manner. It is, of course, concave where the other is convex and much less strongly arched. The fold is narrow and rather high but does not make its appearance for some distance in front of the hinge and the beak is scarcely distinguishable except as a point of symmetry.

This species finds its nearest ally apparently in *C. hillanus* and at first might be thought to be identical with it. My collection contains a number of specimens, however, which are uniform in character and which differ from *C. hillanus* in being arched less strongly and in such a manner that in the pedicle valve the highest parts are toward the front instead of toward the back while the umbonal swell is lower and the beak less prominent. The sinus is narrower and perhaps deeper, though narrowness conveys the impression of depth.

C. consanguineus is obviously of the group of *C. geinitzianus* but, I believe, is a distinct species. Geinitz's figures suggest that the original specimen may have been somewhat crushed and that the broad sinus, angular at the bottom, may have had that shape through accident. If those characters are

original, *C. consanguineus* differs markedly from *C. geinitzianus*, and if they are accidental, detailed comparisons can not be made. Besides there does in fact seem to be a species that agrees quite closely with Geinitz's illustrations, so that the illustrations are probably faithful and the original specimens not impaired by accident.

Horizon and locality: Delaware Mountain formation; Victoria Peak, Diablo plateau, Van Horn quadrangle, Texas.

Chonetes victorianus n. sp.

Figures 1–3

Shell rather small, somewhat pentagonal in outline, very convex with a deep narrow fold and sinus, marked by rather coarse and sharply defined radial lirae.

The pedicle valve is distinctly transverse and widest at the hinge which, however, is not much extended. Below the hinge the outlines at the sides are at first nearly straight with a very general convergence forward. They curve increasingly as they pass downward and rather sharply round the antero-lateral angles while the anterior outline again is almost straight or slightly indented at the middle. Owing to the umbonal parts, which project strongly, the outline as a whole approaches the pentagonal. The convexity is extremely high and dome-like. It is interrupted down the middle by a relatively deep, narrow sinus, so that the vault is divided into two rather strongly rounded ridges having a long steep descent on the outward side to the small, oblique, ill-defined auricles. In conformity with these characters the beak is strongly incurved and the umbonal parts project unusually far beyond the hinge line. The cardinal spines are large, numerous and closely arranged, as many as 7 pores, where spines have been broken off can be counted on one specimen on each side of the beak.

The radial lirae are actually fine but for the genus and for the size of the shell they are relatively coarse. They bifurcate several times in sequence so as to form more or less conspicuous fascicles and the sculpture has the appearance of beginning in the umbonal region with a few plications which subdivide and spread out forward. The "hollow spines" or pore-like openings on the lirae are rather large and rather numerous.

C. victorianus, of course, recalls the eastern shell described as *C. verneuilianus* by Norwood and Pratten. That name has been used many times in paleontologic literature and not always, perhaps, for the same species. The species from Missouri that I cited from so many localities as *C. verneuilianus* is not as highly arched or flexed into so deep a fold and sinus as *C. verneuilianus* appears to be either from the figures given by Norwood and Pratten or the later ones given by Meek. Compared with specimens from Missouri, *C. victorianus* is much more inflated, it has a much deeper fold and sinus, and is marked by much sharper and coarser radial lirae. Thus it appears less to resemble these specimens of *C. verneuilianus* than *C. verneuilianus* as figured. I seem not to have available specimens that can be identified with assurance as *C. verneuilianus* but if such a comparison were possible it would, in view of the wide difference in space, time and faunal association, almost certainly show that *C. victorianus* differed also in its specific characters.

Horizon and locality: Delaware Mountain formation; Victoria Peak, Diablo plateau, Van Horn quadrangle, Texas.

Camarophoria inaequalis n. sp.

Figures 10–13

Shell rather small, strongly transverse, widest below the middle, subtriangular in outline, rather gibbous.

Pedicle valve moderately convex with a rather broad and fairly deep sinus. The sinus is almost flat across the bottom with rather steeply ascending sides. Longitudinally the sinus makes a rather strong curve but a line down the lateral parts of the valve would be nearly straight. These gradually rise above the sinus (or the sinus is gradually depressed) so that they are most elevated where they terminate and from the two points overlooking at the sinus they arch obliquely to the lateral margins. The lateral parts of the shell are much shorter than the part comprising the sinus which projects beyond them in a linguiform manner. At their termination the sinus occupies rather more than one-third of the entire width. The beak, though missing in my specimens, is presumably pointed, sub-erect and rather prominent.

The costae in the sinus are slender and sharply rounded; those on the lateral slopes are larger and more lax. In the type specimen the sinus contains 7 plications and the lateral slopes 4 each with a fifth suggested, the fourth also being short and rather indistinct. The slender plications in the sinus can be distinguished at points farther back than the larger ones of the lateral slopes but all of them fail to reach the umbonal portion.

The brachial valve is gibbous, strongly arched from back to front and likewise strongly arched from side to side. The longitudinal curvature is more pronounced down the lateral parts than down the middle so that the lateral slopes fall away from the fold leaving it a feature though well defined, not very strongly elevated. It is distinctly less elevated than the sinus is depressed; it occupies somewhat less than one-third the width, (only the upper surface being considered) and it is shorter than the lateral slopes which extend appreciably beyond it.

The plications, like those of the pedicle valve, are narrow and sharp on the fold but much larger and somewhat weaker on the lateral slopes. Eight plications occur on the fold, the first apparently branching off from the second and the 8th from the 7th. On the lateral slopes the plications number three with two others of scarcely appreciable strength. As in most shells of this genus, the plications here are irregular in their development, some branching from others as just mentioned and some, on the other hand, attaining normal strength only to disappear farther on.

The foregoing description is based on the type specimen. A second specimen from the same locality agrees with the typical one in its essentials very closely. The fold has only seven plications instead of eight but the first and seventh are branches of the second and sixth in a manner precisely similar. The lateral slopes have three plications of much larger size than those on the median part with one or two others scarcely distinguishable toward the sides. A third specimen shows more noteworthy differences; the fold has seven plications of which the first branches from the second but not the seventh from the sixth. In this specimen the lateral slopes bear almost as numerous, almost as slender and almost as strong plications as the fold.

There are probably seven of these on each side, although the sixth is short and small and the seventh is only obscurely suggested.

Horizon and locality: Delaware Mountain formation, North Apache canyon, Diablo plateau, Texas.

Camarophoria hueconiana n. sp.

Figures 14–21

Shell small, sub-triangular, with the length decidedly less than the greatest width which occurs well forward.

Pedicle valve rather convex and over the median part very strongly arched from back to front. Beak strongly incurved. Umbonal region somewhat inflated. Beginning about one-third the shell length from the beak the median part of the valve becomes depressed into a broad sinus which toward the front occupies about one-half of the entire width if the bounding slopes are included in the sinus. The lateral areas on either side of the sinus are relatively narrow and strongly rounded. The sinus bears 4 rather slender but fairly strong costae which make their appearance well back toward the umbo, before the sinus is an appreciable feature. The lateral slopes are marked by 4 costae of about the same character as those in the sinus though they cannot be traced as far backward and the final costae on each side are very short.

The brachial valve is rather strongly convex, strongly arched from back to front and more strongly arched from side to side. The fold, including its sloping sides, occupies about one-half of the entire valve toward the front but is not strongly elevated except close to the anterior margin. In the front view the fold and sinus are rather high, sharply defined and save for their sloping sides rather flat. The fold bears 5 costae and the lateral slopes 4 each, the costae on the lateral slopes being somewhat conspicuously different from those on the fold for the latter are fairly stout and equal in size to the grooves between them whereas the costae on the lateral slopes are slender and distinctly narrower than the grooves. They appear narrow and also weak, the final one on each side being quite faint.

The foregoing description is drawn up from the typical specimen. Other specimens show certain variations from the characters just noted. The fold may have only 4 plications instead of 5, though this seems to be rare, or it may have 6 which is more common than 4, but decidedly less common than 5. Where 6 occur, the supernumerary costa as a rule is an offshoot from the normal ones; in fact in one specimen on which 5 costae are found only 3 so to speak, are original, the two lateral ones being offshoots from the primary costae to which they are adjacent. Again the contrast between the costae of the lateral slopes and those of the fold is rarely as conspicuous as it is in the type specimen.

The only American species of *Camarophoria* that can be regarded as belonging to the same group with *C. hueconiana* is *C. venusta* but the differences between the two are so obvious as scarcely to need discussion.

Horizon and locality: Upper part of the Hueco limestone; spur south of Cerro Alto, Cerro Alto quadrangle, Texas.

Pugnoides mesicostalis n. sp.

Figures 22–28

Shell rather large, strongly transverse, subelliptical to subpentagonal in outline.

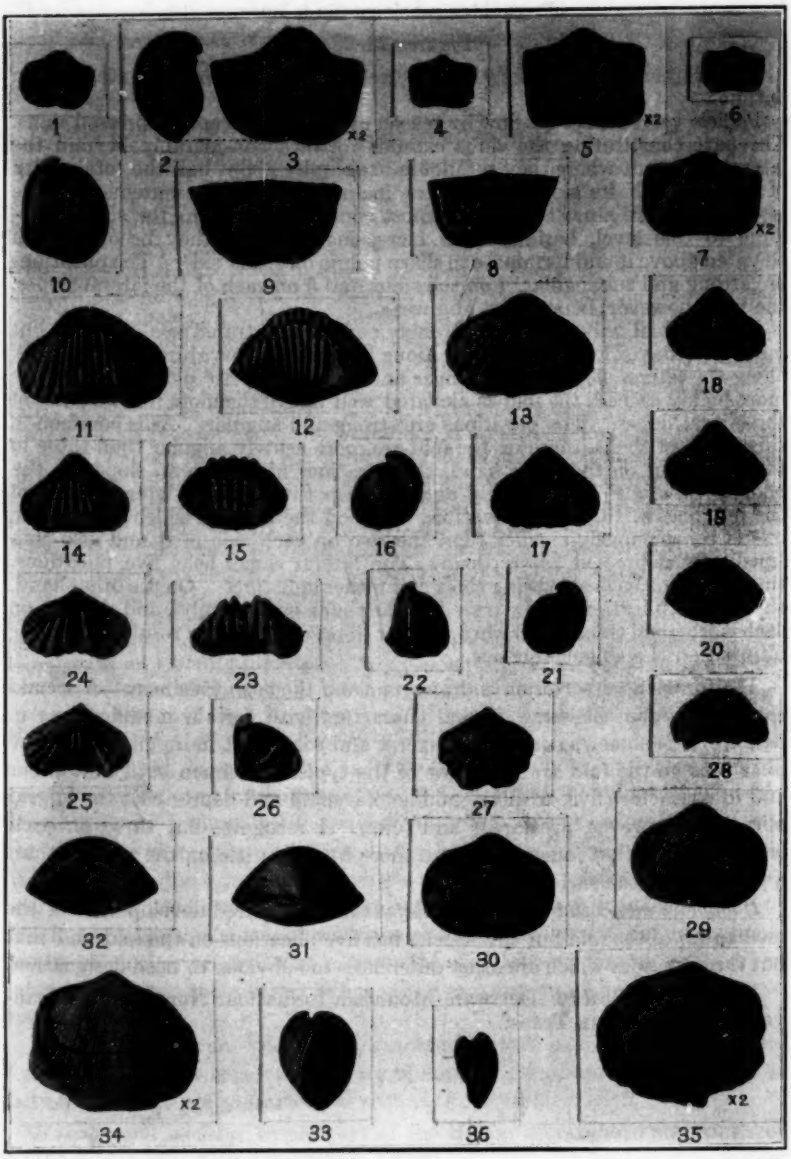
Pedicle valve rather shallow by reason of the large, deeply depressed sinus. The part constituting the sinus extends forward considerably beyond the lateral parts but where they terminate it occupies about half the total width of the valve, if its sloping sides are included in this measurement. The lateral parts are almost flat and almost complanate, and as the sinus is oblique to this level, besides being increasingly arched, they become much elevated above it and terminate in sharp points on either side. The plications are strong and subangular, 4 on the sinus and 5 on each of the lateral slopes, the fifth, however, being short and weak.

The brachial valve is rather strongly convex in a transverse direction but much less convex longitudinally. Along the fold the curvature is very gentle; along the lateral slopes it is stronger and as the sides are oblique, as well as more highly arched, the fold is elevated well above the more remote parts of the lateral slopes. The plications are strong and angular. As is common in this group of shells, those on the fold are more acutely angular than those in the sinus, but on the other hand, the plications of the lateral slopes on the pedicle valve are more acutely angular than those on the lateral slopes of the brachial valve. Five plications occur on the fold, of which the median one is conspicuously smaller than the two on each side of it and also conspicuously depressed below them. The lateral slopes bear five plications, the last one on each side being short and rather indistinct. On the other hand, the median plication can be traced farther back on the umbo, and the two on each side, when traced backward, appear to merge as if they resulted from the bifurcation of a single large one.

The foregoing description is drawn up from the type specimen. A second specimen shows the same general characters with certain modifications in detail. It is somewhat more transverse and somewhat more gibbous. The plications on the fold are like those of the typical specimen both in number and in character (five, with the middle one small and depressed); the lateral plications, however, are larger and fewer. I recognize but three on each lateral slope with a fourth indicated more by a denticle on the margin than a plication on the surface.

Pugnoides mesicostalis recalls *P. elegans* in the aborted development of the median rib of the fold but this species has five plications on the fold and that but three, besides which are other differences too obvious to need designation.

Horizon and locality: Delaware Mountain formation; North Apache canyon, Diablo plateau, Texas.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DESCRIPTION OF FIGURES

Chonetes victorianus, n. sp. (p. 410).

Figs. 1–3. The type specimen, a pedicle valve, natural size and enlarged × 2. Delaware Mountain formation, Victoria Peak, Van Horn quadrangle, Texas.

Chonetes quadratus, n. sp. (p. 407).

Figs. 4–7. Two cotypes, both pedicle valves, natural size and enlarged, × 2. Delaware Mountain formation, Victoria Peak, Van Horn quadrangle, Texas.

Chonetes consanguineus, n. sp. (p. 409).

Figs. 8–9. Two cotypes, both pedicle valves. Delaware Mountain formation, Victoria Peak, Van Horn quadrangle, Texas.

Camarophoria inaequalis, n. sp. (p. 411).

Figs. 10–13. Four views of the typical specimen. Delaware Mountain formation, North Apache Canyon, Diablo Plateau, Texas.

Camarophoria hueconiana, n. sp. (p. 412).

Figs. 14–21. Different views of two specimens, the cotypes. Hueco limestone (upper part), spur south of Cerro Alto, Cerro Alto quadrangle, Texas.

Pugnoides mesicostalis, n. sp. (p. 413).

Figs. 22–25. Four views of the type specimen.

Figs. 26–28. Three views of a somewhat different specimen. Delaware Mountain formation, North Apache Canyon, Diablo Plateau, Texas.

Schizophoria hueconiana, n. sp. (p. 406).

Figs. 29–33. Five views of the type specimen.

Figs. 34–36. Three views of another specimen (figs. 34 and 35), enlarged × 2. Hueco limestone, Marble Canyon, Van Horn quadrangle, Texas.

BOTANY.—*A new variety of henequen without prickles.*[1] LYSTER H. DEWEY, U. S. Department of Agriculture.

Henequen, *Agave fourcroydes* Lemaire, was introduced into Cuba from Yucatan about 1840. Plants descending from that early introduction, propagated chiefly by bulbils and suckers—rarely from seeds—have been used since 1900 in developing large plantations equipped with modern fiber-cleaning machines. These plants, like those of Yucatan, have prickles 1 to 4 mm. long and 10 to 20 mm. apart on the margins of the leaves. The prickles are hooked either upwardly or downwardly and often in both directions on the same leaf.

In 1926, Mr. George H. Simons, President of Compañia Hispano-Americana de Henequen, S. A., found among the plants cultivated by that company at Nuevitas, Cuba, a plant without prickels on the margins of the leaves. The numerous suckers of this plant also had leaves free or nearly free from marginal prickles. This form, originating as a mutation in one plant and continued in its progeny is regarded as worthy of varietal distinction.

[1] Received September 17, 1929.

Agave fourcroydes espiculata Dewey, var. nov.

Mature plant at flowering time with trunk up to 1 m. below the leaves; leaves gray-green, slightly darker on face, 120 to 150 cm. long, 12 to 14 cm. wide, nearly cylindrical and about 3.5 cm. in diameter at base, terminal spine dark chestnut, shiny on upper half, duller and finely pitted under a lens on lower half, 20 to 30 mm. long and 6 mm. in diameter, slightly decurrent on back and margins, with shallow or indistinct groove on lower half of face; margins of leaf straight and smooth, without even rudimentary prickles, or rarely 2 to 6 small prickles on some of the leaves; flower stalk about 5 m. high, bearing dense clusters of erect flowers on very short pedicels on the forked ends of nearly horizontal branches; flowers with green ovary about 20 mm. long, constricted at top, and light yellow perianth about 30 mm. long, the lobes slightly longer than the tube; stamens with curved anthers exserted about the length of the lobes beyond the perianth and style with 3-lobed stigma extending beyond the stamens.

Type in the U. S. National Herbarium, no. 1,411,770, collected at Nuevitas, Cuba, January 14, 1929, by George H. Simons.

This variety differs from the typical form of the species in the absence of prickles on the margins of the leaves and in the narrower and more nearly cylindrical base of the leaves. The flowers are like those of the species except slightly smaller.

The mother plant is exceptionally vigorous, having produced about 350 leaves as compared with about 250 for average henequen plants, and 120 suckers which are also exceptionally vigorous. The mother plant was about 15 or 16 years old at the time of flowering in May, 1929. The variety is of economic interest because the smooth-margined leaves may be handled more easily than those with prickles.

SCIENTIFIC NOTES AND NEWS

At the recent meeting of the American Chemical Society at Minneapolis it was stated that the membership of the Society is now well over 17,000. Papers were presented before seventeen different divisions, and a number of other group meetings were held.

Miss J. L. V. McCord, long connected with the Geological Survey and Librarian for the past 21 years, has retired. Mr. Guy E. Mitchell succeeds Miss McCord as Librarian.

A. Nelson Sayre has been appointed assistant geologist in the Water Resources Branch of the Geological Survey.

C. H. Birdseye has resigned as Chief Topographic Engineer of the Geological Survey and J. G. Staack has been appointed to this position.

Glenn S. Smith has resiged as division engineer of the Atlantic Division of the Topographic Branch and Albert Pike has been appointed to this position. Mr. Smith will continue part time service in the Survey.

Doctor HERBERT FRIEDMANN has been appointed Curator of Birds in the National Museum. Doctor Friedmann held a National Research Council fellowship from 1923 to 1926, during which time he investigated the habits of parasitic birds. He is especially interested in the birds of Africa.

Professor A. S. HITCHCOCK, Agrostologist of the Department of Agriculture, who has been studying the grasslands of Kenya Colony, left Mombasa toward the end of September, and expects to reach Washington about the middle of November.

MONTROSE W. HAYES, recently in charge of the St. Louis station of the Weather Bureau, has been selected as chief of the river and flood division of the Bureau and reported for duty in Washington early in October.

On September 23rd, Mr. NEIL M. JUDD, Curator of American Archeology in the National Museum, returned to Washington after four months' field work in Arizona under the auspices of the National Geographic Society. Mr. Judd's investigations this summer were chiefly concerned with the collection of beams from prehistoric pueblo ruins. There is but a single remaining gap in the "tree ring" chronology being erected by Doctor A. E. DOUGLASS, of the University of Arizona, and it is hoped with this summer's beam collection to bridge this gap and thus make possible the dating of Pueblo Bonito and other pre-Spanish ruins of the Southwest. Although under the general direction of Mr. Judd, the National Geographic Society's 1929 excavations were directly supervised by Mr. L. L. HARGRAVE, of the Museum of Northern Arizona, Flagstaff, and Mr. E. W. HAURY, of the University of Arizona, Tucson. A review of the material resulting from the expeditions is now being made by Doctor Douglass and a report is anticipated in the near future.

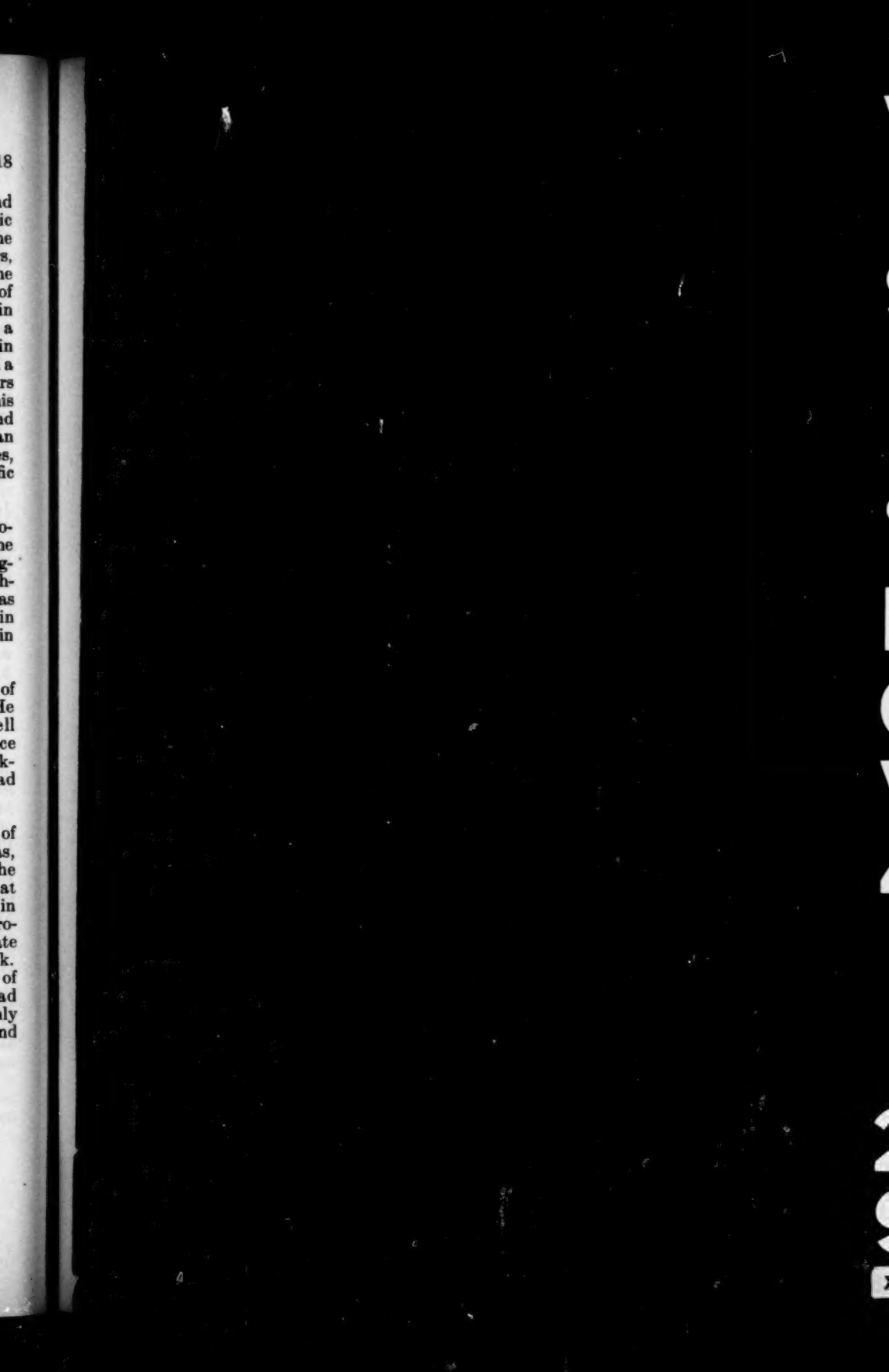
Obituary

Dr. GEORGE PERKINS MERRILL, Head Curator of the Department of Geology, U. S. National Museum, and a member of the ACADEMY, died suddenly from heart disease on August 16th, 1929, in a railroad station at Auburn, Maine. He was born in Auburn May 31, 1854, and attended the University of Maine, where he received the degrees of B.S. in 1879, M.S. in 1883, and Ph.D. in 1889. He also took work at Wesleyan and Johns Hopkins Universities, and in 1917 was awarded the honorary degree of Sc.D. by George Washington University. After teaching for brief periods at various institutions, he was appointed an assistant in the Department of Geology of the Museum in 1881, gradually advancing until he became Head Curator of the Department in 1897. In 1890–91 he was a lecturer in geology at the University of Maryland, and from 1893 to 1915 Professor of Geology and Mineralogy at George Washington University. He belonged to a number of scientific societies; was a fellow of the Geological Society of America (Vice President, 1920), a member of the Geological Society of Washington (President, 1906), and was elected a member of the National Academy of Sciences in 1922. He also received from this organization the J. Lawrence Smith gold medal in recognition of his work on meteorites.

Doctor Merrill's scientific work covered many branches of geology and related sciences, his special fields of interest being the non-metallic minerals, meteorites, and the history of geology, especially in America. In the first of

these fields, he wrote several books, including: Stones for Building and Decoration; Rocks, Rock Weathering and Soils; and The Non-metallic Minerals, Their Occurrence and Uses. He made detailed studies of the chemistry and mineralogy of meteorites, published in numerous briefer papers, and one comprehensive work, Handbook and Descriptive Catalogue of the Meteorite Collections in the U. S. National Museum (1916). One mineral of frequent occurrence in stony meteorites, which had been misinterpreted in various ways by other workers, was shown by his microchemical tests to be a phosphate unknown in terrestrial rocks, and it was appropriately named in his honor, merrillite. In the course of his historical studies, he assembled a remarkably comprehensive collection of autographs and portraits of workers in geology and related sciences. From these studies there resulted his History of American Geology, History of American State Geological and Natural History Surveys, and The First One Hundred Years of American Geology. Shortly before his death he completed The Story of Meteorites, Part 1 of Minerals from Earth and Sky, Volume 3 of the Smithsonian Scientific Series.

Dr. CHARLES WILLIAMSON RICHARDSON, Emeritus Professor of Laryngology and Otology in George Washington University and a member of the ACADEMY, died August 25, 1929. He was born in Washington, D. C., August 22, 1861, studied at the University of Pennsylvania and at George Washington University, taking the degree of Doctor of Medicine in 1884. He was given the honorary degree of Doctor of Science by George Washington in 1921. Dr. Richardson wrote numerous papers dealing with problems in otorhinology and laryngology, in which fields he was an eminent specialist.

Dr. FRANK HURLBURT CHITTENDEN, Senior Entomologist, Bureau of Entomology, Department of Agriculture, died September 15, 1929. He was born in Cleveland, Ohio, November 3, 1858, graduated from Cornell University in 1881, and received the honorary degree of doctor of science from the University of Pittsburgh in 1904. He was an authority on truck-crop insects, and on the taxonomy of certain groups of Coleoptera, and had contributed extensively to entomological literature.

Dr. EDWIN EMERY SLOSSON, Director of Science Service and a member of the ACADEMY, died October 15, 1929. He was born in Sabetha, Kansas, June 7, 1865, and attended the University of that state, receiving the degrees of B.S. in 1890 and M.S. in 1892. He also took advanced work at the University of Chicago, leading to the receipt of a Ph.D. in 1903; and in 1923 the same University awarded him the degree of LL.D. He was Professor of Chemistry at the University of Wyoming and Chemist at the State Experiment Station from 1891 to 1904, resigning to take up literary work. In 1921 he was appointed to the position which he held up to the time of his death. Combining, to an unusual degree, literary skill with a broad knowledge of science, his numerous essays and books have been highly successful in bringing to the layman an appreciation of scientific ideas and of the significance of modern advances in science.